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On the Coded Packet Relay Network in the Presence of Neighbors: Benefits of Speaking in a Crowded Room

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***Abstract*—This paper studies the problem of optimal use of a relay for reducing the transmission time of data packets from a source to a destination using network coding. More importantly, we address an effect that is typically overlooked in previous studies: the presence of active transmitting nodes in the neighborhood of such devices, which is typical in wireless mesh networks. We show that in systems with a fair medium access control mechanism (MAC), the use of a relay in a crowded medium brings forth considerable and unforeseen improvements, including up to 3.5x gains in terms of throughput compared to using only the direct link in some of our examples, and a considerable extension of the operating region where using a relay is beneficial. The problem is formulated as a Markov Decision Process (MDP) and numerical results are provided comparing simple, close–to–optimal heuristics to the optimal scheme.**

I. Introduction

The broadcast nature of wireless channels, which allows potentially all nodes in the transmission range to receive the packets, has opened a series of potential advantages and challenges in the use of the transmission medium in wireless networks. In fact, exploiting relay nodes to improve performance of a single transmission link has been the focus of research under different contexts, but particularly at the physical (PHY) layer, for several decades. The advent of network coding (NC) [1] offers a key mechanism to exploit the benefits of a relay with packet–level interactions, instead of tailored PHY layer mechanisms, by providing a richer, controllable and throughput optimal alternative to simply repeating the same data packet from the relay. The use of random linear network coding (RLNC) [2] allows the system to improve performance requiring minimal if any coordination between relay and source. Nodes need only combine data packets linearly in a finite field using coding coefficients drawn uniformly at random from the elements of the field.

Recent results focused on the coded erasure relay channel, e.g., [3], [4], have studied both performance benefits as well as where and how much to code in this simple network. [5], [6] investigate the problem of relaying from a physical layer perspective for multiple users and multiple relays. Taking a step further, PlayNCool [7], [8] provided more practical mechanisms for exploiting relays in a wireless mesh network to reinforce links chosen by an underlying routing mechanism. This contrasted with previous approaches, e.g., [9], [10], which focused on defining their own routing scheme. Another interesting feature of [7], [8] is the potential increase in performance due to neighboring nodes.

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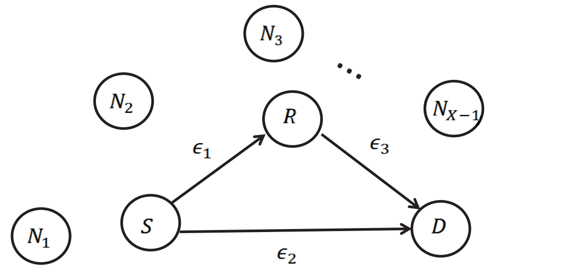

Fig. 1: A coded packet relay network with neighbors. All nodes share a single transmission channel.

Inspired by the flow analysis and simulations in [7], [8], we aim at determining the optimal performance in terms of total transmission time to solve the scenario depicted in Fig. 1. Our problem focuses on determining the optimal transmission policy to send M data packets from S to D with the help of R and in the presence of $X-1$ active neighbors sharing the same channel. To the best of our knowledge, this is the first in–depth analytical work looking at this problem and one that will allow us to understand if the heuristics proposed in [7], [8] have close–to–optimal performance. Seeking to understand the effect of neighboring nodes on the performance of the packet erasure relay channel, we make the following contributions:

- **Mathematical Analysis:** we model the problem as a Markov Decision Process (MDP). The cost of packet transmission is defined as the number of time slots that is used to send packets plus the number of time slots that the sender needs to wait in order to have a time slot allocated to it. For simplicity, we assume a dynamic TDMA medium access control (MAC), although random access can also be modeled with our approach albeit with additional complexity.
- **Numerical Results and Comparison to Heuristics:** we calculate the expected completion time for different scenarios, e.g, different number of neighbors, different number of packets, different erasure probabilities of the links between source, relay, and destination. These results shows two key and counter–intuitive results. First, that the judicious use of a relay can provide gains of up to 3.5x with respect to the use of the direct link. Second, that the operating region where the relay provides benefits can be significantly extended with respect to the result in [3]

when the coded relay network is in the presence of active neighbors. Finally, a comparison between the optimal results obtained by MDP and the simulation results of PlayNCool [7], [8] is provided showing that PlayNCool provides a close–to–optimal solution for many scenarios.

II. Problem Statement

We consider a network that consists of one source (S), one relay (R), and one destination (D), in the presence of $X-1$ neighbors that also use the same channel to transmit data packets (See Fig. 1). A time–slotted system is assumed with only one transmission per time slot and no collisions. We assume a fair time division multiple access (TDMA) medium access control that allows for immediate dynamic allocation of resources based on the nodes' requirements. This TDMA scheme makes the assignments based on transmission rounds, where each active node can transmit at most one time. We model losses between S, R, and D as independent, time–invariant erasure channels, where there is some probability of losing each transmitted packet. The probability of packet loss is given by ϵ_1, ϵ_2, and ϵ_3 for the links from S to R, from S to D, and from R to D, respectively. The source is assumed to have M data packets to transmit, namely, packets $p_1, p_2, ..., p_M$. When transmitting, the source and the relay send linear combinations of the contents of their buffer following the rules of RLNC. For the source, this means generating coded packets by linear combinations of the M original packets using randomly chosen coding coefficients $\alpha_{1,k}, ..., \alpha_{M,k}$ to create the k-th coded packet, i.e., $\sum_{i=1}^{M} \alpha_{i,k} p_i$. The coding coefficients are selected independently and randomly from a Galois field of size q, i.e., $GF(q)$, using a uniform distribution over the elements of the field. For this work, it is assumed that q is large enough so that any RLNC packet received from the source is independent from previously received packets with very high probability. However, this is not the case for transmissions between R and D because they may share common linear combinations.

Assuming that the relay can help the source by transmitting coded packets, when is this beneficial? If the erasure probability of the link between S, D, ϵ_2, is larger than the erasure probability of the link between R, D, ϵ_3, it is clear that it is beneficial to ask for help. If ϵ_2 is lower than ϵ_3, the potential benefits are not as clear. In fact, [3] showed that $\epsilon_3 > \epsilon_2$ for the specific case of no neighbors ($X-1=0$, in our case) is optimally solved without a relay. This means that in an isolated environment with no interference the relay should not be used as it is stealing wireless resources from the source. However, the use of a relay may become beneficial in the presence of neighbors (interferers) in the environment. Although the relay may be using resources that could be allocated to the source, it is inherently providing a larger share among all nodes if the MAC distributes resources equally among the nodes. The heuristics proposed in [7] suggest that this improvement is possible, but the gap between the heuristics and the optimal policy is not addressed. Having these questions in mind, we are interested in finding a packet transmission policy that can minimize the total cost of finishing the transmission of M packets from S to D with/without the help of a relay and in the presence of $X-1$ active neighbors. The cost is defined as the number of active neighbors that use the same channel to transmit plus the number of time slots that we use to transmit packets toward destination.

III. Our MDP Solution to the Problem

We model the problem as an MDP. At each time step, the process is in some state s, and the system may choose any action a that is available in state s. The process continues in the next time step by randomly moving into a new state s' and adding a corresponding cost to the cost of system. For determining the optimal policy, we assume that we have a Genie system (GS) in which the state information of the network is available per time slot and thus, it can help us to choose the best action. In the following, we specify the state, possible actions, and transition probabilities in our model.

A. State Definition:

We define a degree of freedom (DOF) as a linearly independent combination of the original packets. Using this definition, each state is defined by a triplet $s(i_1, i_2, c)$, where i_1 is the number of DOF of the received packets at relay, R, i_2 is the number of DOF of the received packets at destination D. c is the number of DOF of R and D combined, i.e., the dimension of the common knowledge between R and D.

B. Possible Actions:

We define actions a_1, a_2, a_3, a_4 as possible ways of transmitting a packet in the network of Fig. 1 as follows.

- Action a_1: broadcast from S to R, D.
- Action a_2: unicast from R to D.
- Action a_3: first, broadcast from S to R, D, then unicast from R to D in two consecutive time slots.
- Action a_4: do not transmit.

C. Transition Probabilities:

The possible states to which state (i_1, i_2, c) can transit to with non–zero probability depends on the action that we choose and also the total knowledge ($\mathcal{K} = i_1 + i_2 - c$) that is available to both relay and destination at time t. We define $I_{x \in X}$ as an indicator function, which is one when $x \in X$ and zero otherwise. In order to calculate the transition probabilities between different states, we should note that there are two cases where the state of the network does not change, 1) the packet is not received correctly (is erased by the channel), 2) the packet is received correctly but it is not innovative to the set of received packets at destination, in the sense that the received packet is not linearly independent from the previous received packets. The non–zero transition probabilities for 4 possible actions are summarized as follows:

Action a_1 (source broadcast): When the source is broadcasting, there are different possible state transitions. We will explain the more surprising cases, while the rest can be obtained via combinatorial arguments. On the one hand, assuming that the packet is received without erasure at R and D and depending on the total knowledge that is available to both. If the total knowledge is less than M and the packet is not erased by any one of the channels, then the common knowledge between R, D is increased by one since both R, D have received the same packet that is innovative to both of them. If the total knowledge is equal to M and the packet is not erased, the common knowledge between R, D is increased by two. Let us illustrate this with an example. Assuming that $M = 3$ and the set of packets received by R and D until now is P_1, P_3 and $P_2 + P_3$, respectively. The network state is then $s = (2, 1, 0)$. Now assume that source broadcasts $P_1 + P_2 + P_3$, which adds one DOF to R and D. However, the common knowledge is increased by two and the system then transits to a new state $s' = (3, 2, 2)$. On the other hand, if the relay has M DOF, then any new coded packet sent by the source adds one DOF to the destination and increases the common knowledge by one. This is because R already has all DOF needed to decode the original packets and the common knowledge simply equal to the knowledge at D. We now summarize all possible transitions with non–zero probabilities for source broadcasting as

- If $\mathcal{K} < M$, $i_1 < M$, $i_2 < M$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} + \epsilon_1(1-\epsilon_2)$ $I_{(i_1'=i_1,i_2'=i_2+1,c'=c)} + (1-\epsilon_1)\epsilon_2\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c)} +$ $(1-\epsilon_1)(1-\epsilon_2)\ I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+1)}$.
- If $\mathcal{K} = M$, $i_1 < M$, $i_2 < M$:

  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} + \epsilon_1(1-\epsilon_2)$ $I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)} + (1-\epsilon_1)\epsilon_2\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c+1)}$ $+ (1-\epsilon_1)(1-\epsilon_2)\ I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+2)}$.
- If $\mathcal{K} = M$, $i_1 = M$, $i_2 \neq M$:

  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_2\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} + (1-\epsilon_2)$ $I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)}$.
- If $i_2 = M$: $P_{(i_1,i_2,c)\to(i_1,i_2,c)} = 1$.

Action a_2 (unicast from R to D): If the number of DOF at R is equal to the common knowledge of R, D, the relay cannot send a packet to D that adds one DOF to it. On the other hand, if the number of DOF at R is greater than the common knowledge, then the packet sent by R adds one DOF to the set of received packets by D under our high field size assumption. We summarize the transition probabilities as

- If $i_2 < M$, $i_1 > c$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_3\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} + (1-\epsilon_3)$ $I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)}$.
- If $i_2 < M$, $i_1 = c$ or $i_2 = M$: $P_{(i_1,i_2,c)\to(i_1,i_2,c)} = 1$.

Action a_3 (first broadcast, then unicast from R to D): This action includes two consecutive phases and constitutes a combination of a_1 and a_2 occuring in the same transmission round. Starting by state s, first we use broadcast to transit to a new state $\hat{s}$ with probability $p_{s\to\hat{s}}$ and then assuming that the system is in state $\hat{s}$, we calculate the transition probability of transition from $\hat{s}$ to s' using action a_2 as $p_{\hat{s}\to s'}$. Therefore, the transition probability of going from state s to state s' using action a_3 is calculated as $p_{s\to s'} = p_{s\to\hat{s}} \times p_{\hat{s}\to s'}$. Using combinatorial arguments, the transitions are as follows.

- If $\mathcal{K} < M$, $c < i_1 < M$, $i_2 < M-1$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\epsilon_3\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} +$
  $\epsilon_1\epsilon_2(1-\epsilon_3)\ I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)} +$
  $(1-\epsilon_1)\epsilon_2\epsilon_3\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c)} +$
  $[(1-\epsilon_1)\epsilon_2(1-\epsilon_3) + (1-\epsilon_1)(1-\epsilon_2)\epsilon_3] \times$
  $I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+1)} + \epsilon_1(1-\epsilon_2)\epsilon_3 \times$
  $I_{(i_1'=i_1,i_2'=i_2+1,c'=c)} + \epsilon_1(1-\epsilon_2)(1-\epsilon_3) \times$
  $I_{(i_1'=i_1,i_2'=i_2+2,c'=c+1)} + (1-\epsilon_1)(1-\epsilon_2) \times$
  $(1-\epsilon_3)\ I_{(i_1'=i_1+1,i_2'=i_2+2,c'=c+2)}$
- If $\mathcal{K} < M$, $c < i_1 < M$, $i_2 = M-1$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\epsilon_3\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} +$
  $\epsilon_1\epsilon_2(1-\epsilon_3)\ I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)} +$
  $(1-\epsilon_1)\epsilon_2\epsilon_3\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c)} +$
  $[(1-\epsilon_1)\epsilon_2(1-\epsilon_3) + (1-\epsilon_1)(1-\epsilon_2)] \times$
  $I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+1)} + \epsilon_1(1-\epsilon_2) \times$
  $I_{(i_1'=i_1,i_2'=i_2+1,c'=c)}$
- If $\mathcal{K} < M$, $i_1 < M$, $i_2 < M-1$, $i_1 = c$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} +$
  $(1-\epsilon_1)\epsilon_2\epsilon_3\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c)} +$
  $[(1-\epsilon_1)\epsilon_2(1-\epsilon_3) + (1-\epsilon_1)(1-\epsilon_2)] \times$
  $I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+1)} + \epsilon_1(1-\epsilon_2) \times$
  $I_{(i_1'=i_1,i_2'=i_2+1,c'=c)}$
- If $\mathcal{K} < M$, $i_1 < M$, $i_2 = M-1$, $i_1 = c$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} +$
  $(1-\epsilon_1)\epsilon_2\epsilon_3\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c)} +$
  $[(1-\epsilon_1)\epsilon_2(1-\epsilon_3) + (1-\epsilon_1)(1-\epsilon_2)] \times$
  $I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+1)} + \epsilon_1(1-\epsilon_2) \times$
  $I_{(i_1'=i_1,i_2'=i_2+1,c'=c)}$
- If $\mathcal{K} = M$, $c+1 < i_1 < M$, $i_2 < M-1$:
  $P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_1\epsilon_2\epsilon_3\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} +$
  $[\epsilon_1\epsilon_2(1-\epsilon_3) + \epsilon_1(1-\epsilon_2)\epsilon_3]\ I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)}$
  $\epsilon_1(1-\epsilon_2)(1-\epsilon_3)\ I_{(i_1'=i_1,i_2'=i_2+2,c'=c+2)} +$
  $(1-\epsilon_1)\epsilon_2\epsilon_3\ I_{(i_1'=i_1+1,i_2'=i_2,c'=c+1)} +$
  $[(1-\epsilon_1)\epsilon_2(1-\epsilon_3) + (1-\epsilon_1)(1-\epsilon_2)\epsilon_3] \times$
  $I_{(i_1'=i_1+1,i_2'=i_2+1,c'=c+2)} + (1-\epsilon_1)(1-\epsilon_2) \times$
  $(1-\epsilon_3)\ I_{(i_1'=i_1+1,i_2'=i_2+2,c'=c+3)}$
- If $\mathcal{K} = M$, $i_1 = M$, $i_2 < M-1$, $i_1 > c+1$:

$$P_{(i_1,i_2,c)\to(i_1',i_2',c')} = \epsilon_2\epsilon_3\ I_{(i_1'=i_1,i_2'=i_2,c'=c)} + [\epsilon_2(1-\epsilon_3) + (1-\epsilon_2)\epsilon_3]\ I_{(i_1'=i_1,i_2'=i_2+1,c'=c+1)} + (1-\epsilon_2)(1-\epsilon_3)\ I_{(i_1'=i_1,i_2'=i_2+2,c'=c+2)} \quad (1)$$

- If $i_2 = M$: $P_{(i_1,i_2,c)\to(i_1,i_2,c)} = 1$
- If $\mathcal{K} = M$, $i_1 = M$, $i_2 = M-1$:

$$P_{(i_1,i_2,c)\to(i'_1,i'_2,c')} = \epsilon_2\epsilon_3 \; I_{(i'_1=i_1,i'_2=i_2,c'=c)}+ \\ [\epsilon_2(1-\epsilon_3)+(1-\epsilon_2)] \; I_{(i'_1=i_1,i'_2=i_2+1,c'=c+1)} \tag{2}$$

- If $\mathcal{K}=M,\ i_1=c+1,\ i_2=M-1$:

$$P_{(i_1,i_2,c)\to(i'_1,i'_2,c')} = \epsilon_1\epsilon_2\epsilon_3 \; I_{(i'_1=i_1,i'_2=i_2,c'=c)}+ \\ [\epsilon_1\epsilon_2(1-\epsilon_3)+\epsilon_1(1-\epsilon_2)] \; I_{(i'_1=i_1,i'_2=i_2+1,c'=c+1)}+ \\ (1-\epsilon_1)\epsilon_2\epsilon_3 \; I_{(i'_1=i_1+1,i'_2=i_2,c'=c+1)}+ \\ [(1-\epsilon_1)\epsilon_2(1-\epsilon_3)+(1-\epsilon_1)(1-\epsilon_2)]\times \\ I_{(i'_1=i_1+1,i'_2=i_2+1,c'=c+2)} \tag{3}$$

Action a_4 (do not transmit): $P_{(i_1,i_2,c)\to(i_1,i_2,c)}=1$.

D. Cost Function

It is assumed that one transmission is done per time slot. Therefore, every time the source or the relay transmit a packet, they have to wait for $X-1$ time slots to get one time slot assigned for them to transmit their packets again. If both S, R transmit in two consecutive time slots, then the number of time slots that is used is $X+1$ in that transmission round. On the other hand, if only one transmits the number of slots in a round is X. Fig. 2 shows the cost of actions a_1, a_2, a_3. This leads to

$$C(s,a_j,s') = \begin{cases} X, & \forall s \in S \mid s \neq (i_1,M,c), \\ & \quad j \in 1,2 \\ (X+1), & \forall s \in S \mid s \neq (i_1,M,c), j=3 \\ \mathcal{D}, & for\ s=(i_1,M,c), j \in 1,2,3, \\ \mathcal{D}, & \forall s \in S \mid s \neq (i_1,M,c), j=4, \\ 0, & \text{if } s=(i_1,M,c),\ j=4, \end{cases} \tag{4}$$

where $C(s,a_j,s')$ is the cost of transition from state s to state s' by choosing action a_j and S is the set of all possible states. $\mathcal{D}$ is an arbitrary large number that is much greater than X. By defining $\mathcal{D} >> X$, we make sure that the MDP does not choose any one of the actions a_1, a_2, a_3 if the system is in the absorbing states, $s(i_1,M,c)$, and it chooses action a_4 that has the minimum cost. This leads to stopping the process at the absorbing states. We define a single absorbing state in this case as being composed by a set of states of the form (i_1,M,c), where i_1 can change from zero to M.

E. Optimization Algorithm

We use the value iteration algorithm (Bellman equations) [11] to solve the optimization problem and to minimize the total cost of the transmission of M packets. A value function is defined as $V_t : S \to R^+$ that associates to each state s a lower bound on the minimal total cost $V^*(s)$ that should be paid starting from that state. We can summarize the steps to find an optimal policy as

$$V_0(s)=0,\ \forall s \in S, \\ V_{t+1}(s) \leftarrow \min_a E(C(s,a,s')+\zeta V_t(s')), \tag{5}$$

where $E(X)$ shows the expected value of X. This will iterate until the condition $\max_s |V_{t+1}(s)-V_t(s)| < \delta$ is satisfied. t represents the iteration number and $\zeta \in (0,1]$ is called discount factor and used to make sure that the equation converges when t goes to infinity and δ has a very small value greater than zero (e.g. 0.01).

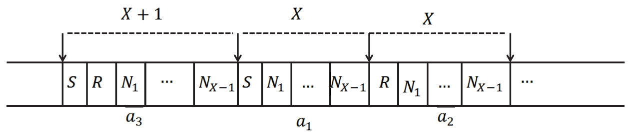

Fig. 2: Cost (required time slots) of three key actions

IV. Numerical Results

A. Comparison Schemes

We use two schemes to compare the performance of the relay approaches, namely, the MDP solution and the PlayNCool heuristic [7], [8], with the performance of a non–relay approach in the presence of active neighbors and using RLNC. We also compare the heuristic to the MDP solution to assess its performance compared to the optimal approach.

1) MDP: The MDP scheme is the optimal solution to the problem that we have discussed before and is computed as discussed in Section III.

2) PlayNCool: The PlayNCool scheme uses a simple heuristic to transmit packets opportunistically. The source starts broadcasting until the relay receives a reasonable number of the DOF (but not enough to decode) before it starts to send. When the relay starts sending, it will also listen to transmissions from the source to gather additional DOF. Both relay and source transmit RLNC packets until the destination receives enough DOF to decode. The number of broadcast transmissions before relay starts sending, r, depends on the erasure probabilities of the channels. This means that the relay makes decisions based only on knowledge of its own state and channel statistics, but not on the receiver state. If the relay is close to source and far from destination, i.e., $(1-\epsilon_1)\times\epsilon_2 > 1-\epsilon_3$, r is calculated as $r=\frac{1}{(1-\epsilon_1)\epsilon_2}$. If the relay is closer to destination, i.e., $(1-\epsilon_1)\times\epsilon_2 \le 1-\epsilon_3$, the number of transmissions before relay starts sending is calculated as [7]:

$$r = \frac{-M.C(\epsilon_1,\epsilon_2,\epsilon_3)}{D(\epsilon_1,\epsilon_2,\epsilon_3)-(1-\epsilon_2).C(\epsilon_1,\epsilon_2,\epsilon_3)}, \tag{6}$$

where $C(\epsilon_1,\epsilon_2,\epsilon_3)=(-1+\epsilon_3+\epsilon_2-\epsilon_1.\epsilon_2)$ and $D(\epsilon_1,\epsilon_2,\epsilon_3)=(2-\epsilon_2-\epsilon_3).(\epsilon_2-\epsilon_1.\epsilon_2)$.

B. Comparison Scenarios

We use the C++ KODO library [12] to simulate the PlayNCool protocol and compare it with the optimal MDP solution. We consider three scenarios to analyze the effect of different parameters of the network on the gain of coded packet relay networks: a) M, X are fixed while ϵ_i is varied, b) ϵ_i and M are fixed while X is varied, and c) ϵ_i, X are fixed while M is varied. The gain in the presence of $X-1$ active neighbors is defined as the completion time of sending M packets from S to D without relay (CT_{WR}) divided by the completion time of a relay approach (CT_R) that is calculated by simulation or the MDP solution:

$$Gain = \frac{CT_{WR}}{CT_R}. \tag{7}$$

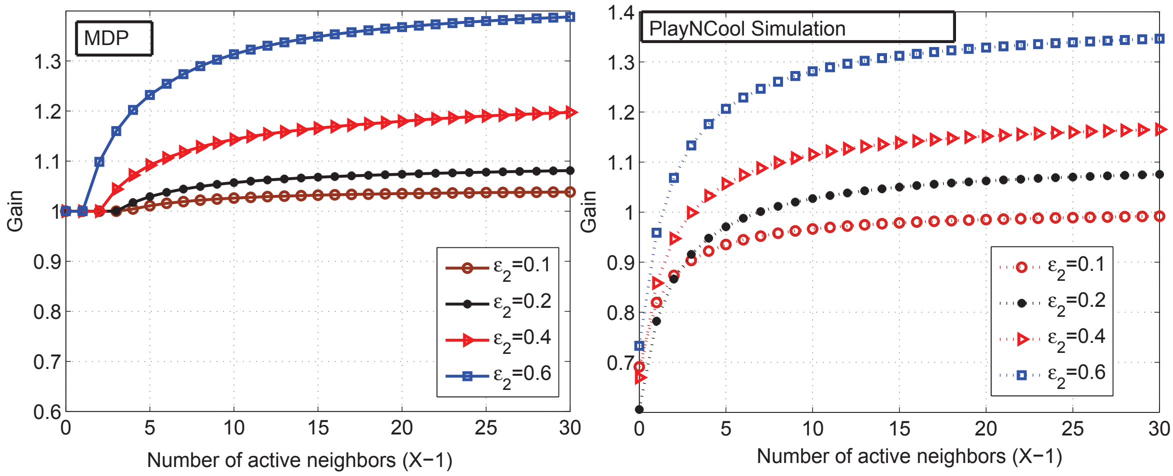

Fig. 3: Comparison between MDP, and PlayNCool simulation for $\epsilon_1 = 0.2, \epsilon_3 = 0.8, M = 10$ and different number of active neighbors

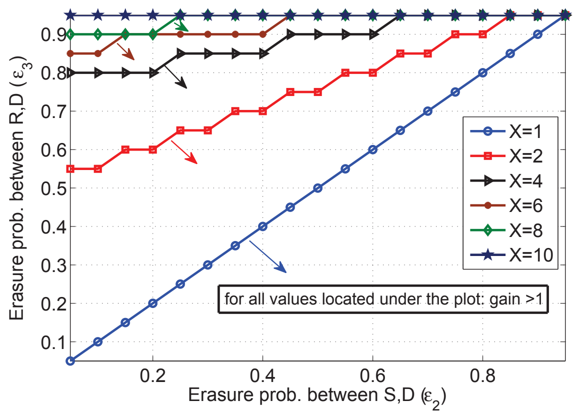

Fig. 4: The map of possible area of getting benefit of using relay for $\epsilon_1 = 0.2, M = 10$ and different values of $\epsilon_2, \epsilon_3, X$: pairs of (ϵ_2, ϵ_3) under the curve of X provide gain> 1, i.e., there is a gain of using the relay

1) The effect of erasure probabilities: We investigate different scenarios to validate our claim that a *crowded room* (i.e., active neighbors) allows the relay to provide additional benefits. First, we consider the case where the erasure probability of the channel between R and D is more than the erasure probability of channel between S and D, which was shown in [3] to require no relay to achieve optimal performance (no other active nodes). Fig. 3 illustrates that the use of the relay can be beneficial if there are active neighboring nodes in the system. This corresponds to cases with a gain larger than 1. The MDP solution demonstrates that even a small number of neighbors is sufficient to make the use of a relay attractive.

Fig. 3 also shows that PlayNCool does not provide a good solution for this region until there is a large number of active nodes, suggesting that improvements are needed in the heuristics of [7], [8]. However, when enough neighboring nodes are active the performance of PlayNCool comes closer to the performance of optimal MDP solution. Fig. 3 shows that even a poor link between R and D ($\epsilon_3 = 0.8$ in this case) can help in decreasing the time to complete the transmission of $M = 10$ packets by around 40 %.

In order to have a better understanding of the effect between neighboring nodes in the usefulness of a relay, we illustrate the operating region where the relay provides benefits. This useful operating region for the erasure probabilities of the links between S, D (ϵ_2) and R, D (ϵ_3) is defined for each X value as the area under the curve (pointed by an arrow) in Fig. 4. In other words, the relay provides gains for pairs of (ϵ_2, ϵ_3) that are located under the curve for each X. The curves were calculated using the MDP solution for $X = 1, 2, 4, 6, 8, 10$ and different pairs (ϵ_2, ϵ_3). Fig. 4 for the case of $X = 1$, which is the same as having no neighbors in the network, confirms the result in [3]. That is, if $\epsilon_2 < \epsilon_3$ there is no gain of using relay. By increasing the number of active neighbors, we increase the region where we get benefits of using a relay. Even a single neighbor, i.e., $X = 2$, provides a significant increase in the useful operating region. For $X = 10$, essentially any pair (ϵ_2, ϵ_3) benefits of using a relay, as shown in Fig. 4. Finer grained results can be computed using a larger number of points, but the key result still holds: the presence of neighbors makes the relay useful in a wider range of channel conditions.

Second, we consider the case where the link between R, D is better than the link between S, D. We assume that $\epsilon_2 = 0.8$ and there are $X - 1 = 5$ active neighbors in the network. Fig. 5 shows a similar experiment for the case where $\epsilon_3 = 0.3$ and ϵ_1 is changed for both $M = 10$ and $M = 30$ packets. Fig. 5 shows that by increasing the erasure probability of the channel between R, D, the gain of relay approaches decreases but it is still greater than one. This means that even if the channel between R and D is not substantially better than the one between S to D, the presence of active neighbors makes the use of a relay beneficial to speed up the packet transmission process. Also, Fig. 5 shows that by increasing the value of M, the gap between the gain calculated by the MDP and the simulation is decreased. This is explained because PlayNCool assumes that R is always sending innovative packets to D, while this is not always true as we have shown in the MDP analysis. By increasing the number of packets, the probability of sending innovative packets increases and therefore, the performance of PlayNCool is closer to the MDP solution.

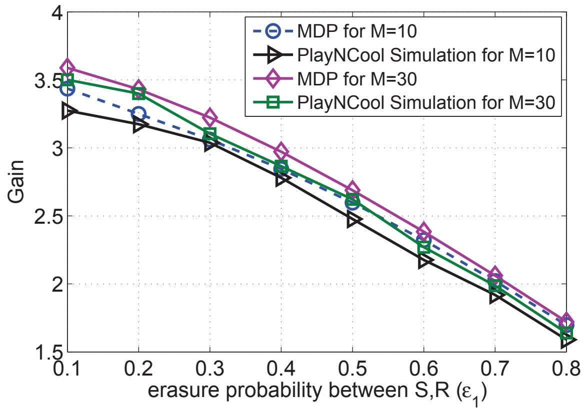

Fig. 5: Gains of MDP and PlayNCool simulation for $\epsilon_2 = 0.8, \epsilon_3 = 0.3$, $X-1=5$, and different values of ϵ_1 and M

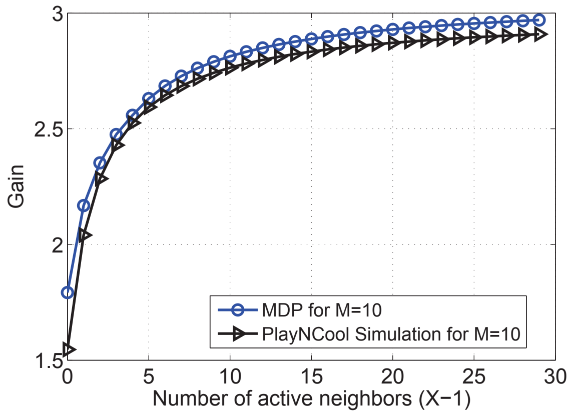

Fig. 6: Gains of MDP and PlayNCool simulation for $\epsilon_1 = 0.3, \epsilon_2 = 0.8, \epsilon_3 = 0.5$, $M = 10$ packets, and different number of active neighbors $(X-1)$

2) The effect of number of active neighbors: We assume that $\epsilon_1 = 0.3, \epsilon_2 = 0.8, \epsilon_3 = 0.5$ and $M = 10$ for the network depicted in Fig. 1. In order to see the effect of network traffic on the gain of the relay approaches, we change the number of active neighbors that are competing to access the same channel. Fig. 6 presents the gain of PlayNCool protocol with the gain of the optimal MDP solution for 0 to 29 neighbors. By increasing the number of interfering nodes, the gain of using a relay approach increases. Fig. 6 shows that the gap between the PlayNCool heuristic and optimal MDP solution is below 10%, which is quite impressive since PlayNCool does not assume perfect knowledge of the system state.

3) The effect of number of packets (M): We assume that $\epsilon_1 = 0.3, \epsilon_2 = 0.8, \epsilon_3 = 0.6$ and $X-1=5$ active neighbors. We change the number of packets that are transmitted from S to D. Fig. 7 compares the gains of PlayNCool and the MDP solution with respect to the direct link for M changing from 5 to 30. By increasing the number of packets, the gain of both MDP and PlayNCool increases while their gap decreases.

V. Conclusion

We proposed a Markov Decision Process model to determine the optimal policy to minimize the transmission time of M packets from a source to a destination in the presence of

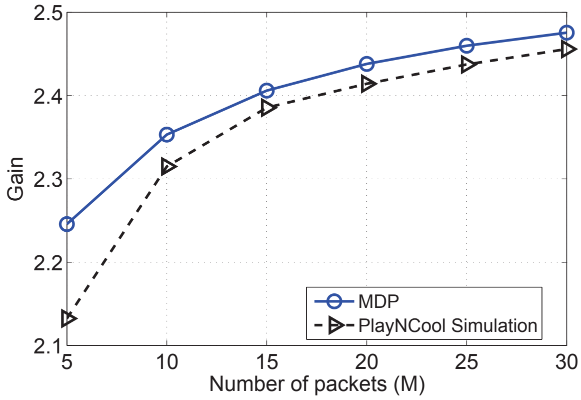

Fig. 7: Gains of MDP and PlayNCool simulations for $\epsilon_1 = 0.3, \epsilon_2 = 0.8, \epsilon_3 = 0.6, X-1=5$ and different M

$X-1$ active neighbors by using RLNC and a relay approach. We compared the performance of the optimal MDP solution to that of the PlayNCool protocol proposed in [7], [8] in terms of the completion time for a transmission of M packets for different scenarios, e.g, different number of active neighbors, different number of packets, and different channel conditions. Our results show that PlayNCool is able to achieve the close–to–optimal performance, when the number of packets is large. More importantly, we showed that using a relay in the presence of active neighbors is beneficial even if the channel from relay to destination is not better than the channel between source and destination. Future work will consider the effects of asymmetric coding and modulation schemes for transmission from source and relay, which can increase even more the usefulness of the relay as well as more complex topologies, e.g., multi–hop scenarios, sharing of relay by multiple flows.

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